THERMOELASTIC DESTRUCTION OF TRANSPARENT MEDIA BY LASER RADIATION

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It is shown that thermoelastic stresses may be the mechanism for the laser destruction of homogeneous transparent media having high thermal and elastical strength, at least during the initial stage of this destruction. Threshold values of the radiation density required for thermoelastic destruction are calculated.

One of the most probable mechanisms for the laser destruction of a transparent brittle medium is the appearance of thermoelastic stresses. Studies which have appeared on the thermoelastic mechanism for the effects of radiation on a material have dealt with either the simplest cases or the effects of "gigantic" laser pulses [1-7]. Khodyko has discussed certain problems involving thermoelastic stresses.

We report here a solution of the spatial problem of the temperature field in a medium heated by laser radiation and the quasistatic problem of the thermoelastic stresses.

We consider an infinite plate of a transparent material of thickness H at an initial temperature T_0. At $t = 0$, radiation of intensity $q_0(t)$ is incident perpendicular to the plate surface on a circle of radius R; the radiation intensity is uniform over this circle (Fig. 1). We call the irradiated volume "region 1" and everything else $r > R$ "region 2." The flux density at any cross section z = const is independent of r and is given by the Bouguer law

$$q(t,\ z) = q_0(t)\exp(-kz).$$

The heat conductivity equations and the boundary conditions for regions 1 and 2 are

$$\begin{gathered}\frac{\partial T_1}{\partial t} = a\left(\frac{\partial^2 T_1}{\partial r^2} + \frac{1}{r}\frac{\partial T_1}{\partial r} + \frac{\partial^2 T_1}{\partial z^2} - \frac{1}{\lambda}\frac{\partial q(t,\ z)}{\partial z}\right),\\ \frac{\partial T_2}{\partial t} = a\left(\frac{\partial^2 T_2}{\partial r^2} + \frac{1}{r}\frac{\partial T_2}{\partial r} + \frac{\partial^2 T_2}{\partial z^2}\right).\end{gathered} \tag{1}$$

At $t = 0$, we have $T_1 = T_2 = T_0$; at $z = 0, H$, we have $\partial T_1/\partial z = \partial T_2/\partial z = 0$; at $r = 0$, we have $\partial T_1/\partial r = 0$; as $r \to \infty$, $T_2 = 0$; at $r = R$, we have

$$T_1 = T_2 \text{ and } \frac{\partial T_1}{\partial r} = \frac{\partial T_2}{\partial r}. \tag{2}$$

By the successive use of a Laplace time transformation and a finite Fourier cosine transformation with respect to the coordinate z, we find a solution for the boundary-value problem (1)-(2) for the Laplace transform of the temperature to be

$$\begin{aligned}\bar{T}_1(r,\ z,\ p) = \frac{T_0}{p} &+ \frac{\bar{q}_0(p)\,[1-\exp(-kH)]}{\lambda H \dfrac{p}{a}}\left[1-\sqrt{\frac{p}{a}}\,RK_1\left(\sqrt{\frac{p}{a}}\,R\right)I_0\left(\sqrt{\frac{p}{a}}\,r\right)\right]\\ &+ \frac{2}{H}\sum_{n=1}^{\infty}\frac{\bar{q}_0(p)\,k^2\,[1-\exp(-kH)\cos n\pi]}{\lambda m^2\left(k^2+\dfrac{n^2\pi^3}{H^2}\right)}\,[1-mRK_1(mR)\,I_0(mr)]\cos\frac{n\pi z}{H};\end{aligned}$$

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$$\overline{T}_2(r,\ z,\ p) = \frac{T_0}{p} + \frac{\overline{q}_0(p)\,R\,[1-\exp(-kH)]}{\lambda H\sqrt{\frac{p}{a}}}\ I_1\left(\sqrt{\frac{p}{a}}\,R\right)K_0\left(\sqrt{\frac{p}{a}}\,r\right)$$
$$+\frac{2}{H}\sum_{n=1}^{\infty}\frac{\overline{q}_0(p)\,k^2R\,[1-\exp(-kH)\cos n\pi]}{\lambda m\left(k^2+\frac{n^2\pi^2}{H^2}\right)}\ I_1(mR)\,K_0(mr)\cos\frac{n\pi z}{H}. \tag{3}$$

It is extremely cumbersome to carry out the inverse Laplace transformation for arbitrary kH. For the transparent materials discussed in this paper ($k < 1$), and for relatively thin materials, we have $kH \ll 1$. Accordingly, the sum of the series in Eqs. (3) can be neglected, with an error evaluated to be no worse than 1%. We can carry out the inverse Laplace transformation of these transforms using the Duhamel theorem on transform convolution (see, e.g., [8]); the result is a common expression for the temperature in regions 1 and 2:

$$T(r,\ t) = T_0 + \frac{ak}{\lambda}\int_0^t\left[q_0(\tau) - \frac{R}{2a\tau}\,q_0(t-\tau)\int_0^r \exp\left(-\frac{y^2+R^2}{4a\tau}\right)I_1\left(\frac{yR}{2a\tau}\right)dy\right]d\tau. \tag{4}$$

This solution is valid for any duration of the irradiation. For typical laser-irradiation times, the parameter p is large, so an asymptotic expansion for the modified Bessel functions [8] can be used to invert Eq. (3) and to find the approximate solution

$$T_i(r,\ t) = T_0 + \frac{q_0akt}{2\lambda}\,[1+(-1)^{i+1}] + \frac{q_0akt}{2\lambda}\sqrt{\frac{R}{r}}\,(-1)^i\left\{\frac{1}{8}\left[\left(6+\frac{3r^2-R^2}{rR}\right)\right.\right.$$
$$\left.+\frac{|R-r|^2}{6at}\left(22+\frac{3r^2-R^2}{rR}\right)\right]\Phi^*\left(\frac{|R-r|}{2\sqrt{at}}\right)+\frac{1}{6\sqrt{\pi}}\left[(-1)^{i+1}\sqrt{at}\left(\frac{3}{R}+\frac{1}{r}\right)\right.$$
$$\left.\left.-\frac{|R-r|}{4\sqrt{at}}\left(22+\frac{3r^2-R^2}{rR}\right)\right]\exp\left(-\frac{|R-r|^2}{4at}\right)\right\}\quad (i=1,\ 2). \tag{5}$$

The error involved in the replacement of exact solution (4) by the approximation (5) does not exceed the error due to the discarding of the sum, i.e., 1%.

Illustrative calculations have been carried out for polycrystalline sapphire, K-8 silicate glass, and polymethylmethacrylate, which differ significantly in strength and thermal properties. The calculations were carried out for a typical laser pulse 2 msec in duration, approximated by the smooth function (where t is in milliseconds and E is in joules)

$$E(t) = \frac{340\,t}{1+0{,}8t},\ \text{and}\ q_0(t) = \frac{1}{\pi R^2}\,\frac{dE(t)}{dt}.$$

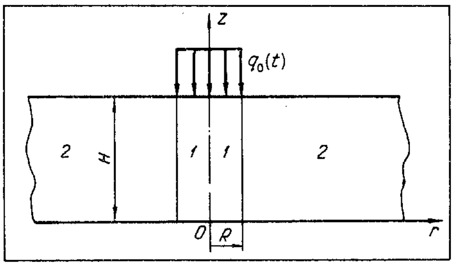
This approximation also holds under "burst" conditions, since the temperature at any point remains essentially constant during the interval between bursts (3-10 μsec).

Figure 2 shows the temperature distribution in 1 mm plates for the case R = 0.2 mm, which corresponds to "sharp" focusing of the radiation, at times t* corresponding to the elastic yield point of the material. The temperature dependence of the properties of the materials was taken into account [9, 10]. The effect of thermal conductivity on the temperature distribution is quite evident from these curves.

We now seek the thermoelastic stresses themselves. Since the displacement velocity is small (~10 cm /sec), we will solve the quasistatic problem. Since, when there is weak absorption, the temperature given by Eq. (4) is independent of z, i.e., the plate is thin, all the stresses and displacements are essentially independent of z. It is a good approximation to consider this a planar stress state (see, e.g., [11]); because of the symmetry about the beam axis, the stresses and displacements do not depend on the angular coordinate.

Substituting the stress equation in terms of the displacements,

$$\sigma_{rr}^{(i)} = \frac{E}{1-\nu^2}\left[\frac{dU_r^{(i)}}{dr}+\nu\frac{U_r^{(i)}}{r}-(1+\nu)\,\alpha\theta_i(r,\ t)\right],$$
$$\sigma_{\varphi\varphi}^{(i)} = \frac{E}{1-\nu^2}\left[\frac{U_r^{(i)}}{r}+\nu\frac{dU_r^{(i)}}{dr}-(1+\nu)\,\alpha\theta_i(r,\ t)\right]$$
$$(i=1,\ 2)$$



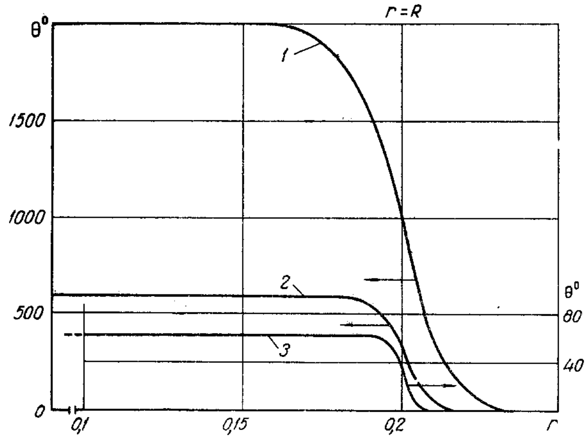


Fig. 1 Fig. 2

Fig. 1. Scheme for irradiating the transparent material: 1) irradiation region; 2) region not irradiated.

Fig. 2. Temperature distribution $\theta(r, t)$ (R = 0.2 mm): 1) sapphire (t = 0.22 msec); 2) K-8 glass (t = 1.1 msec); 3) polymethylmethacrylate (t = 0.05 msec).

into the equilibrium equation,

$$\frac{d}{dr}\sigma_{rr}^{(i)} + \frac{1}{r}[\sigma_{rr}^{(i)} - \sigma_{\varphi\varphi}^{(i)}] = 0,$$

we find the following equations for the displacements

$$\frac{d}{dr} r^3 \frac{d}{dr} \frac{U_r^{(i)}}{r} = (1+\nu)\alpha r^2 \frac{d\theta_i}{dr} \quad (i = 1,\ 2) \tag{6}$$

where the boundary conditions are

$$\begin{aligned} &r = 0 \quad U_r^{(1)} = 0,\\ &r \to \infty \quad \sigma_{rr}^{(2)} = 0,\\ &r = R \quad U_r^{(1)} = U_r^{(2)} \text{and} \sigma_{rr}^{(1)} = \sigma_{rr}^{(2)}. \end{aligned} \tag{7}$$

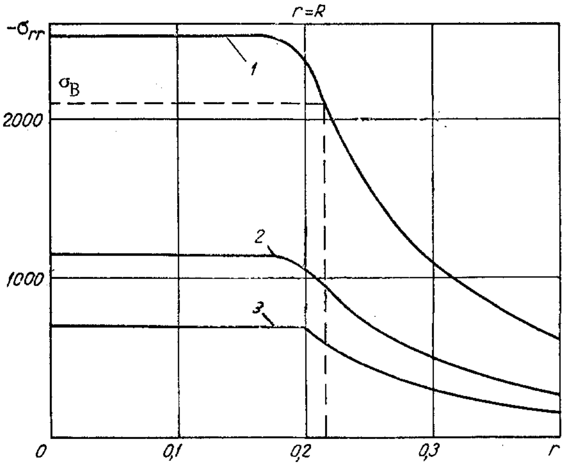
The solution of the boundary-value problem (6)-(7) is

$$\begin{aligned} \sigma_{rr}^{(1)} &= -\frac{\alpha E}{r^2}\int_0^r r\,\theta_1(r)\,dr,\\ \sigma_{rr}^{(2)} &= -\frac{\alpha E}{r^2}\left[\int_0^R r\,\theta_1(r)\,dr + \int_R^r r\,\theta_2(r)\,dr\right],\\ \sigma_{\varphi\varphi}^{(i)} &= -\left[\sigma_{rr}^{(i)} + \alpha E\,\theta_i(r)\right] \quad (\text{t fixed; i} = 1,2). \end{aligned} \tag{8}$$

Substituting the expression for the temperature from Eqs. (4) or (5) into (8), we find the distribution of the stress $\sigma_{rr}^{(i)}$ or $\sigma_{\varphi\varphi}^{(i)}$ at any instant of time.

Figures 3 and 4 show the stress fields for sapphire, K-8 glass, and polymethylmethacrylate. The values of α and E used were averages over the temperature range through which each material was heated (see, in particular, [12]). The curves show that the stress σ_{rr} is everywhere compressional, while the stress $\sigma_{\varphi\varphi}$ is also compressional in the radiation region but tensile in the region r > R, having a maximum near the boundary r = R.

Let us evaluate the possibility of destruction in these examples. When only normal σ_{rr} and $\sigma_{\varphi\varphi}$ are acting, brittle destruction occurs when one of the stresses reaches the corresponding strength σ_B [13]. The curves in Figs. 3 and 4 show that the destructive stresses are attained right at the start of the laser pulse – near t = 0.22 msec at r ≤ 0.22 mm for σ_{rr} and r ≤ 0.46 mm for $\sigma_{\varphi\varphi}$. In glass, destruction occurs near t = 1.1 msec for r ≤ 0.204 for $\sigma_{\varphi\varphi}$. The thermoelastic stresses do not reach the strength limits in polymethylmethacrylate.



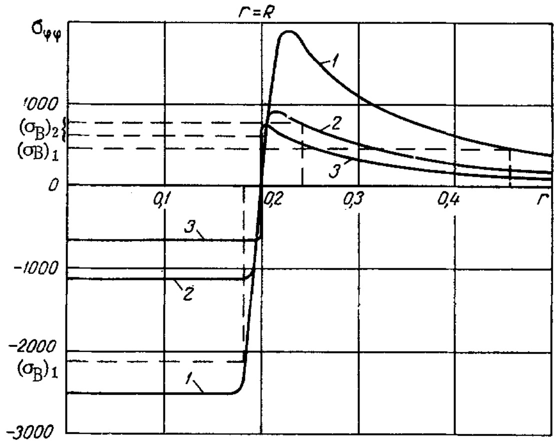


Fig. 3 Fig. 4

Fig. 3. Distribution of the stress σ_{rr}, MN/m^2 (R = 0.2 mm): 1) sapphire (t = 0.22 msec); 2) K-8 glass (t = 1.1 msec, $\sigma_{rr}\cdot 10$); 3) polymethylmethacrylate (t = 0.05 msec, $\sigma_{rr}\cdot 100$).

Fig. 4. Distribution of the stress $\sigma_{\varphi\varphi}$, MN/m^2 (R = 0.2 mm): 1) sapphire (t = 0.22 msec); 2) K-8 glass (t = 1.1 msec, $\sigma_{\varphi\varphi}\cdot 10$); 3) polymethylmethacrylate (t = 0.05 msec, $\sigma_{\varphi\varphi}\cdot 100$).

It has thus been shown through a solution of the heat-conductivity and thermoelastic-stress problem that a pulse of (free-generation) laser radiation concentrated in a small area will produce thermoelastic stresses sufficient to destroy polycrystalline sapphire or K-8 glass. The threshold radiation flux density ε_t calculated for sapphire is 10^4 J/cm^2 (an average over the pulse having a flux $\bar{q}_t$ equal to roughly $0.5 \cdot 10^7$ W/cm^2) and $\varepsilon_t \approx 2.7 \cdot 10^4$ J/cm^2 for the glass ($\bar{q}_t \approx 2.5 \cdot 10^8$ W/cm^2). The irradiation of polymethylmethacrylate at a radiation flux density of $\varepsilon = 1.3 \cdot 10^4$ J/cm^2 ($\bar{q} = 2.6 \cdot 10^8$ W/cm^2) does not cause its destruction by the mechanism of thermoelastic stress. Since polymethylmethacrylate begins to soften at high energy densities, it is practically impossible to destroy by the thermoelastic-stress mechanism.

In a real material, other mechanisms may operate (thermal explosion at nonuniformities, shock waves, and electrical breakdown), depending on the medium and parameters of the incident radiation, and will have an independent effect on the kinetics and ultimate results of the interaction. In homogeneous, transparent media having high thermal and electrical strength, thermoelastic stresses will be the basic cause of laser destruction, at least during the initial stage of the irradiation. The formation of the initial cracks, which act as inhomogeneities in the thermoelastic mechanism leads to an abrupt increase in absorption of light energy and to the development of local thermal explosions resulting in the formation of shock waves, thus causing an expansion of the destroyed zone and the rate of its formation. Strictly speaking, the thermoelastic cracking of a material in the overall laser-irradiation process is governed by the properties of the transparent medium and the parameters of the light pulse; as the examples of sapphire and K-8 glass show, this cracking may occur outside the radiation zone.

NOTATION

r, z	are the cylindrical coordinates;
t	is the time;
h	is the sample thickness;
R	is the radius of the irradiation zone;
T	is the temperature;
$\theta(r, t) = T(r, t) - T_0$	is the instantaneous and initial temperature difference;
ε	is the radiation energy density;
q	is the radiation flux;
$\bar{T}(r, z, p) = \int_0^\infty T(r, z, t)\exp(-pt)dt$	is the Laplace transform of the temperature;
p	is the Laplace transformation parameter;
$I_0(\tau), I_1(\tau), K_0(\tau), K_1(\tau)$	are the modified Bessel functions;
$\Phi^*(x) = (2/\sqrt{\pi})\int_x^\infty \exp(-\tau^2)d\tau$	is the error function;
$m^2 = p/a + n^2\pi^2/H^2$	is the parameter in Eqs. (3);

k	is the absorption coefficient;
a	is the thermal diffusivity;
λ	is the thermal conductivity;
α	is the coefficient of linear expansion;
ν	is the Poisson ratio;
E	is the Young's modulus;
U_r	is the displacement along the r coordinate;
σ_{rr}, $\sigma_{\varphi\varphi}$	are the normal stresses.

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